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The Latest Arrival Hub Location Problem

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The traditionally studied hub location problems in the literature pay attention to flight times but not to transient times spent at hubs for unloading, loading, and sorting operations. The transient times may constitute a significant portion of the total delivery time for cargo delivery systems. We focus on the minimization of the arrival time of the last arrived item in cargo delivery systems and develop a model that correctly computes the arrival times by taking into account both the flight times and the transient times. Nonlinear and linear integer formulations are given and computational results are provided. The effects of delays on the system performance are analyzed.

(Hub Location; Minimax; Latest Arrival)

Hub location problems arise when it is desirable to consolidate and disseminate flows at certain centralized locations called hubs. Most applications arise in airline passenger travel (Toh and Higgins 1985), cargo delivery (Kuby and Gray 1993, O'Kelly 1998), and message delivery in computer communication networks (Klincewicz 1998). The basic structure of hub location problems can be described as follows: There are n nodes that generate or absorb flows. To take advantage of economies of scale, and possibly because of other managerial considerations, the flows from origins to destinations are consolidated and then disseminated at hubs. The main problem involves determining the locations of hubs and the allocation of nodes to hubs so as to carry the cross-traffic to minimize a cost function. The cost of the hub-to-hub portion of the journey is discounted by a factor α ($0 \leq \alpha \leq 1$) to account for the economies of scale resulting from the increased traffic between hubs. The cost function to be minimized can be minisum, minimax, or covering type (Campbell 1994a). The model that has received the most attention in the literature is the p -hub median problem, which is the minisum version (O'Kelly 1986, 1987; Aykin 1995; Campbell

1994a, 1996; Ernst and Krishnamoorthy 1996, 1998; Skorin-Kapov et al. 1996). The minimax version, the p -hub center problem, has recently been studied by Kara and Tansel (2000) (see also Campbell 1994a for an initial formulation and O'Kelly and Miller 1991 for the special case $p = 1$). The hub covering problem has been defined by Campbell (1994a) and is also studied by Kara and Tansel (1999). The interested reader may consult the survey papers by O'Kelly and Miller (1994), Campbell (1994b), and Bryan and O'Kelly (1999) for further information.

Most existing formulations of hub location problems are based on an $n \times n$ symmetric cost matrix $C = [c_{ij}]$, which is assumed to satisfy the triangle inequality. In the minisum problem, c_{ij} is interpreted to be the cost of carrying one unit of flow from i to j . In the minimax and covering versions, it is more appropriate to interpret c_{ij} to be the travel time between i and j as these versions seem to be more appropriate models for cargo delivery systems where certain deadlines on delivery time must be met. Our focus in this paper is on cargo delivery, so we use the time-based interpretation of the c_{ij} s.

When time is of concern, one must pay attention to all components of the total delivery time, which includes not only the flight times but also the transient times spent at hubs between flights (Sigafos and Easson 1988, Iyer and Ratliff 1990, O'Kelly and Miller 1991). A closer look at the operations of an overnight delivery system attests to the fact that the transient times at hubs constitute a significant portion of the total delivery time. The typical overnight delivery firm picks up packages from customers at a local station by 7:30 p.m. with a promise to deliver them to their destinations by 8:30 a.m. the next morning. Each incoming package at the local station is labeled (e.g., fragile, hazardous, flammable) and assigned a bar code that includes the zip code of the destination. The processed units are loaded onto an aircraft and are delivered to the hub that serves that local station. There are three major operations at any hub: unloading the arriving aircraft, sorting the cargo, and loading the departing ones. The packages that are unloaded from arriving aircraft are fed into a conveyor system that is equipped with manual or automatic bar code readers. The packages on the conveyor system are sorted according to their zip codes by bar code readers at the feeder lines that read the zip code information. Sorted packages are then routed to the specific area of the hub where they can be reloaded on the correct cargo containers. The outgoing aircraft is ready to depart when all the cargo for its destination is loaded. If a departing aircraft from a hub is destined to go to a nonhub city, then it is unloaded at the local station of its final destination and the unloaded packages are delivered to the consignees by 8:30 a.m. An aircraft that goes from a hub to another hub goes through the unloading, reloading, and the associated sorting/routing operations at the second hub to have its cargo delivered to the final destination cities that are serviced from that hub.

As is evident from the above description, cargo may spend a considerable amount of time at a hub during the process of unloading, sorting, handling, and reloading. The loading operation cannot be completed until all incoming cargo that will be loaded on an aircraft have been received. This results in additional waiting time for units that have arrived earlier. The

additional waiting time may be quite large, depending on how late the latest arriving unit is. This paper proposes a new hub location model that takes into account the transient times at hubs in addition to the flight times. Even though the transient times at hubs are an integral part of the total journey time, the existing hub location models in the literature do not pay attention to this component of the delivery time. The proposed model fills a gap in this respect. We refer to the proposed model as the *latest arrival hub location problem*. Minimax, covering, and minisum versions for the latest arrival hub location problem can be distinguished depending on the structure of the objective function. Our focus is on the minimax version. We study various aspects of this problem, including model development, linearization, computational aspects, time zones, and analysis of delays in departure times.

We now give an overview of the paper. Section 1 is devoted to the development of the latest arrival hub location problem. Initially, a combinatorial formulation is given in implicit form. Then, the model development is carried out to derive appropriate algebraic expressions for the transient times. This results in a combinatorial formulation in explicit form. In §2, we prove the NP-hardness of the minimax latest arrival hub location problem. In §3, we give a nonlinear mixed-integer programming formulation for the minimax latest arrival hub location problem, then linearize it. Additionally, we give an alternative linearization, which is directly derived from the combinatorial formulation with a certain switch of viewpoint. We incorporate the effects of different time zones into our model in §4. In §5, we report our computational results on the performance of the linear integer model using CPLEX 5.0 based on 60 instances of the standard CAB data set. In §6, we derive expressions for the available slacks in hub departure times and analyze the effects of delays on the system performance. Related what-if questions are also discussed in this section. The paper ends with concluding remarks in §7.

1. Model Development

Suppose we are given n cities numbered $1, \dots, n$, with $t_{ij}(=t_{ji})$ denoting the flight time between cities i

and j . Let $T = [t_{ij}]$ be the $n \times n$ matrix of flight times and assume T satisfies the triangle inequality. The speed of delivery from a hub to a hub is generally different from the speed of delivery between a nonhub and a hub city due to possible differences between modes of transportation or the types of vehicles used. We reflect this difference by means of a parameter $\alpha > 0$. That is, if i and j are both hub cities, then the travel time between them is taken to be αt_{ij} , where $\alpha < 1$ corresponds to faster delivery and $\alpha > 1$ corresponds to slower delivery. The model and all the analytical and computational results that we obtain from it are valid regardless of the value of α . We assume that there is a positive flow, $w_{ij} > 0$, from every origin i to every destination j . We term this assumption the *full cross-traffic assumption*. This seems to be a reasonable assumption for cargo delivery systems. Let r_i be the ready time of the outgoing cargo from city i . Although the packages at a given city i are collected at different times during the day, they can all be assigned a common ready time, r_i , which is the flight departure time from city i . Let $N = \{1, \dots, n\}$ and let H be a subset of N that specifies the locations of hubs with $|H| = p$ ($1 \leq p < n$).

It is possible to distinguish two types of service policies: single assignment and multiassignment. In single assignment, a given node is served from a single hub that handles both outgoing and incoming units of that node. In multiassignment, a given node is served from a set of hubs. In this case, the total traffic originating and ending at a given node is subdivided into parts where each part is handled through a different hub that is assigned to the node under consideration. Observe that allowing multiassignment includes the possibility of assigning each node to exactly one hub and hence any multiassignment model always yields at least as good an objective value as its corresponding single-assignment version. This fact has been observed and used in hub median location research and has led to more efficiently solved mixed-integer programming formulations of hub median problems than the corresponding single-assignment models (Ernst and Krishnamoorthy 1996, 1998). In the p -hub median problem, the multiassignment model can easily be obtained from the

corresponding single-assignment model by a relaxation of the zero/one constraints on the assignment variables. The same conclusion cannot be drawn for the latest arrival hub location problem. This is because there are various complications in the computations of departure times from hubs in the multiassignment case that are not present in the single-assignment case. The additional complications arise from the fact that the "index independence" property, which is a salient feature of the single-assignment problem, does not hold for the multiassignment case because the multiassignment policy must keep track of *which part* of the cargo of a given node must be serviced from which hub. Accordingly, a simple relaxation of the zero/one requirements on the assignment variables does not lead to the corresponding multiassignment model. With these considerations, we focus on the single-assignment problem. This assumption is reasonably well justified in practice as most cargo delivery firms seem to have a tendency to use the single-assignment policy to take advantage of the simplified service structure and the associated administrative and managerial benefits.

Let $a(i) \in H$ be the single hub to which node i is assigned. Let $a = (a(1), \dots, a(n)) \in H^n$ denote any assignment vector (H^n is the n -fold Cartesian product of H with itself). For a specified (location, assignment) pair (H, a) , denote by $T_{ij}(H, a)$ the *total time spent during delivery* from i to j via the hubs $a(i), a(j) \in H$. Thus, the arrival time at j of the units originating at i destined to go to j is $r_i + T_{ij}(H, a)$. The total journey time, $T_{ij}(H, a)$, is the sum of the total flight time and the total transient time; that is,

$$T_{ij}(H, a) = t_{ia(i)} + \alpha t_{a(i)a(j)} + t_{a(j)j} + \tau_{ij}(a(i)) + \tau_{ij}(a(j)), \quad (1)$$

where $\tau_{ij}(a(i))$ and $\tau_{ij}(a(j))$ are, respectively, the transient times at hubs $a(i)$ and $a(j)$ of the units going from i to j . An expression for computing the $\tau_{ij}(\cdot)$ values in terms of the input data will be derived subsequently. The minimax, covering, and minisum versions of the latest arrival hub location problem in *implicit* forms are as follows:

$$(1) \quad \min_{\substack{H \subset N, \\ |H|=p}} \min_{a \in H^n} \max_{i, j \in N} r_i + T_{ij}(H, a), \quad (2)$$

$$(2) \quad \min_{\substack{H \subset N, \\ a \in H^n}} |H| \\ \text{s.t. } r_i + T_{ij}(H, a) \leq \beta \quad \forall i, j \in N, \quad (3)$$

where $\beta > 0$ is a deadline on latest arrival time, and

$$(3) \quad \min_{\substack{H \subset N, \\ |H|=p}} \min_{a \in H^n} \sum_{i, j \in N} w_{ij} T_{ij}(H, a). \quad (4)$$

We now derive an algebraic expression for $T_{ij}(H, a)$. Denote by DT_{pq} the departure time of a flight going from node p to node q . For nonhub origins i , $DT_{ia(i)} = r_i$. To compute $T_{ij}(H, a)$, consider the journey from i to j via the hubs $a(i)$ and $a(j)$. All units going from i to j experience a flight time of $t_{ia(i)}$ during the first segment of this journey. The transient time at $a(i)$ is the departure minus the arrival time of these units. That is,

$$\tau_{ij}(a(i)) = DT_{a(i)a(j)} - (r_i + t_{ia(i)}). \quad (5)$$

To correctly compute the departure time $DT_{a(i)a(j)}$, observe that the aircraft going from $a(i)$ to $a(j)$ transports not only those units that come from i but also the units that come from other nonhub origins that are also serviced from $a(i)$. Note, however, that the triangle inequality on $T = [t_{ij}]$ implies that the aircraft going from $a(i)$ to $a(j)$ does not transport the units that come from other hubs. Accordingly, $DT_{a(i)a(j)}$ is the latest of the arrivals from nonhub origins to $a(i)$. Hence,

$$DT_{a(i)a(j)} = \max_{k: a(k)=a(i)} r_k + t_{ka(i)}. \quad (6)$$

Observe from (6) that $DT_{a(i)a(j)}$ is, in fact, independent of $a(j)$. Hence, the departure time from hub $a(i)$ is the same regardless of which hub the aircraft is flying to. This is true under the assumption of full cross-traffic. If this assumption is not satisfied, (6) must be written as

$$DT_{a(i)a(j)} = \max_{k \in I_{a(i)a(j)}} r_k + t_{ka(j)}, \quad (6')$$

where $I_{a(i)a(j)}$ is the set of origins k such that $a(k) = a(i)$, and $w_{kl} > 0$ for some l for which $a(l) = a(j)$.

The units going from i to j , together with other units that are serviced via the hub pair $(a(i), a(j))$,

experience a common flight time of $\alpha t_{a(i)a(j)}$. The transient time at $a(j)$ for units going from i to j is

$$\tau_{ij}(a(j)) = DT_{a(j)j} - (DT_{a(i)a(j)} + \alpha t_{a(i)a(j)}). \quad (7)$$

Here, $DT_{a(j)j}$ is determined by the latest of the arriving units at $a(j)$ that are destined to go to j . A unit that is destined to go from an arbitrary origin k to node j arrives at $a(j)$ at time $DT_{a(k)a(j)} + \alpha t_{a(k)a(j)}$. Hence,

$$DT_{a(j)j} = \max_{h \in H} DT_{ha(j)} + \alpha t_{ha(j)}. \quad (8)$$

Substituting the right-hand side of (6) for $DT_{ha(j)}$, we have

$$DT_{a(j)j} = \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})]. \quad (9)$$

Observe from (9) that $DT_{a(j)j}$ is, in fact, independent of the destination j . This is again true under the assumption of full cross-traffic. $\tau_{ij}(a(j))$ in Expression (7) is now computable given the values of $DT_{a(j)j}$ in (9) and of $DT_{a(i)a(j)}$ in (6). Substituting the computed forms of $\tau_{ij}(a(i))$ and $\tau_{ij}(a(j))$ in (1) and cancelling out like terms, $T_{ij}(H, a)$ reduces to

$$T_{ij}(H, a) = t_{a(j)j} + \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})] - r_i. \quad (10)$$

Using (10) and dropping the constant term $\sum_{i \in N} w_{ij} r_i$ from the objective function in (13), the *explicit* forms of the minimax, covering, and minisum latest arrival hub location problems, respectively, are as follows:

$$(1') \quad \min_{\substack{H \subset N, \\ a \in H^n, \\ |H|=p}} \min_{a \in H^n} \max_{j \in N} \left(t_{a(j)j} + \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})] \right), \quad (11)$$

$$(2') \quad \min_{\substack{H \subset N, \\ a \in H^n}} |H| \quad \text{s.t. } t_{a(j)j} + \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})] \leq \beta \quad \forall j, \quad (12)$$

$$(3') \quad \min_{\substack{H \subset N, \\ |H|=p}} \min_{a \in H^n} \sum_{j \in N} W_j \left(t_{a(j)j} + \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})] \right), \quad (13)$$

where $W_j = \sum_i w_{ij}$ is the total flow into j .

Note that in the implicit form of the minimax problem defined in (2), the maximum is taken over all

index pairs $i, j \in N \times N$, whereas in the explicit form defined in (11) the maximum is taken on the index $j \in N$ alone. This is justified by the fact that the arrival time at node j is not dependent on the originating index i , i.e., regardless of the ready times, all units from different origins that are destined to go to node j arrive at node j at the same time. Similarly, in the explicit form of the covering problem in (12), one upper-bound constraint is written for each index $j \in N$, whereas in the implicit form in (3) one constraint is written for each index pair $i, j \in N \times N$. This again follows from the fact that the arrival time at node j (the left side of (12)) is not dependent on the originating index i , which is true regardless of the ready times. Similarly, with the omission of the constant term $\sum_{i \in N} w_{ij} t_i$ from the objective function of the minimax problem, the summation of the explicit form in (13) is on the index j alone, whereas the summation is over all index pairs in the implicit form defined by (4).

Hence, the explicit forms reduce the number of terms in the maximand, constraints, or the summation from n^2 to n . This helps to obtain greatly reduced integer programming formulations for these problems. Additionally, the input requirement in (13) is reduced from an $n \times n$ flow matrix $W = [w_{ij}]$ to an n vector (W_1, \dots, W_n) , which is much easier to obtain from the annual inflow records of the local stations rather than having to keep track of the cross-traffic on the entire network. The independence property from the originating indices seem to be a unique feature of the latest arrival hub location problem but is not observable in the traditionally studied hub location problems.

2. The Minimax Latest Arrival Hub Location Problem—Complexity

Our focus in the remainder of the paper is on the minimax version of the latest arrival hub location problem. We first show that this problem is NP-hard. To prove it, take $\alpha = 0$ and $r_i = 0 \forall i \in N$. With $\alpha = 0$, the $\alpha t_{ha(j)}$ term in (11) disappears and the innermost two maximizations output a value $g(H, a) = \max_{h \in H} \max_{k: a(k)=h} t_{kh}$, which depends only on the hub set H and the assignment vector a , but not on the

index j . It is direct now to conclude that, for fixed H , assigning each node j to a hub in H with the minimum travel time is optimal. To see this, let a^* be such an assignment vector. For any other assignment $a \in H^n$, $t_{a^*(k)k} \leq t_{a(k)k} \forall k$. Hence, $g(H, a^*) \leq g(H, a)$ and consequently,

$$\max_{j \in N} t_{a^*(j)j} + g(H, a^*) \leq \max_{j \in N} t_{a(j)j} + g(H, a).$$

It follows that a^* is an optimal assignment. Then, (11) reduces to

$$\min_{\substack{H \subset N, \\ |H|=p}} \max_{j \in N} t_{a^*(j)j} + g(H, a^*) = \min_{\substack{H \subset N, \\ |H|=p}} \max_{j \in N} 2 \left(\min_{h \in H} t_{jh} \right),$$

which is the node restricted p -center problem on a complete graph K_n with arc weights t_{ij} , $i, j \in N$. Hence, the minimax version is a special case of the p -center problem. It is well known that the p -center problem is NP-hard (Kariv and Hakimi 1979), implying that the minimax latest arrival hub location problem is also NP-hard.

3. IP Formulations

In this section, we give integer programming formulations for the minimax version of the latest arrival hub location problem. Recall from (6) that the departure times from a hub h toward all other hubs are the same. Recall also from (9) that the departure times from a hub h toward all cities that are serviced from h are, again, the same. Thus, at any hub h , there are two different departure times: the departure time for aircraft that are destined to go to other hubs, and the departure time for aircraft that are destined to go to nonhub destinations. Let \widehat{DT}_h and DT_h denote these two departure times, respectively. Using (6) and (9), we have

$$\widehat{DT}_h = \max_{k: a(k)=h} (r_k + t_{kh}), \quad (14)$$

$$DT_h = \max_{k \in H} (\widehat{DT}_k + \alpha t_{kh}). \quad (15)$$

Let X_{jk} be a zero/one variable that takes on the Value 1 if node j is assigned to hub k and 0 otherwise. Note that $X_{kk} = 1$ means there is a hub at node k and $X_{kk} = 0$ means there is no hub at node k . An integer

programming formulation for the minimax problem, abbreviated as MML (MiniMaxLatest), is as follows:

(MML) $\min Z$,

$$\text{s.t.} \quad Z \geq (DT_k + t_{jk})X_{jk} \quad \forall j, k, \quad (16)$$

$$\widehat{DT}_k \geq (r_j + t_{jk})X_{jk} \quad \forall j, k, \quad (17)$$

$$DT_k \geq \widehat{DT}_r + \alpha t_{rk} X_{rr} \quad \forall r, k, \quad (18)$$

$$\sum_k X_{jk} = 1 \quad \forall j, \quad (19)$$

$$\sum_k X_{kk} = p, \quad (20)$$

$$X_{jk} \leq X_{kk} \quad \forall j, k, \quad (21)$$

$$X_{jk} = \{0, 1\} \quad \forall j, k. \quad (22)$$

Each node is assigned to exactly one hub by Constraints (19) and (22). Constraint (20) ensures that exactly p hubs are selected. Constraint (21) allows the allocations to be made to hub nodes only. Whenever $X_{jk} = 1$, the right-hand side of (16) gives the arrival time at node j . Hence, (16) forces Z to take on the value of the latest arrival time. Constraints (17) and (18) ensure that \widehat{DT}_k and DT_k take on the intended values (as defined in (14) and (15)) at optimality.

MML is a nonlinear mixed integer program with n^2 zero/one and $2n + 1$ real variables. The number of constraints is $4n^2 + n + 1$. Nonlinearity is due to Constraint (16).

One way to linearize MML is to replace (16) with

$$Z \geq DT_k + t_{jk}X_{jk} - M(1 - X_{jk}), \quad (16')$$

where M is a large positive number. Unfortunately, the computational performance of this linearization is very poor. A less obvious but still correct linearization is to simply drop the last term in (16'), i.e., write

$$Z \geq DT_k + t_{jk}X_{jk} \quad (23)$$

in place of (16). We call this linearization L1. The correctness of this linearization can be justified by observing that any feasible solution to L1 is also a feasible solution to MML, and that any optimal solution to L1 is also optimal for MML. Feasibility can be directly justified. To justify optimality, let $(Z^*, \mathbf{DT}^*, \widehat{\mathbf{DT}}^*, \mathbf{X}^*)$ be an optimal solution to L1. If

this solution is not optimal to MML, then there is a feasible solution $(Z', \mathbf{DT}', \widehat{\mathbf{DT}}', \mathbf{X}')$ to MML with objective value $Z' < Z^*$. It can be shown that the solution $(Z', \mathbf{DT}'', \widehat{\mathbf{DT}}', \mathbf{X}')$, where $DT_k'' = \max_r (\widehat{DT}_r' + \alpha t_{rk} X_{rr}')$ $\forall k$ is a feasible solution to L1 with objective value $Z' < Z^*$. This contradicts the optimality of $(Z^*, \mathbf{DT}^*, \widehat{\mathbf{DT}}^*, \mathbf{X}^*)$.

We now give a second linear model that is directly obtained from the combinatorial formulation by a reinterpretation. For fixed (H, a) , let $A_j(H, a)$ be the common arrival time at node j from all origins. That is, $A_j(H, a) = t_{a(j)j} + \max_{h \in H} [\alpha t_{ha(j)} + \max_{k: a(k)=h} (r_k + t_{kh})]$. Using the auxiliary variable DT_h defined in (15), we also have $A_j(H, a) = t_{a(j)j} + DT_{a(j)}$. It now follows that

$$\max_{j \in N} A_j(H, a) = \max_{h \in H} \left(DT_h + \max_{k: a(k)=h} t_{kh} \right). \quad (24)$$

Hence, we may rewrite the explicit form of the minimax latest arrival hub location problem as

$$\min_{\substack{H \subset N, \\ |H|=p}} \min_{a \in H^n} \max_{h \in H} \left(DT_h + \max_{k: a(k)=h} t_{kh} \right). \quad (25)$$

This form of the combinatorial formulation directly leads to the following linear integer program, (L2):

$$\min Z, \quad \text{s.t.} \quad Z \geq DT_h + \rho_h \quad \forall h \quad (26)$$

$$\rho_h \geq t_{kh} X_{kh} \quad \forall k, h \quad (27)$$

$$(17) - (22),$$

where ρ_h is another auxiliary variable that takes on the value $\max_{k: a(k)=h} t_{kh}$ at optimality. Note that there is no nonlinearity in this new formulation. L2 requires n^2 zero/one and $3n + 1$ real variables. The number of constraints is $4n^2 + 2n + 1$.

Observe that L1 and L2 are essentially the same linear integer programs since ρ_h is just an auxiliary variable and can be removed to convert (26) and (27) to the form $Z \geq DT_k + t_{jk}X_{jk}$, which is nothing but (23). Despite the fact that L1 and L2 have essentially the same mathematical structure, they are obtained out of entirely different considerations. L1 is simply a linearization of the nonlinear model MML, which is the natural model for hub location researchers since it focuses on the analysis of what goes on during the journey from an origin i to a destination j via the

assigned hubs $a(i)$ and $a(j)$. On the other hand, L2 is directly obtained from the combinatorial formulation by a reinterpretation that requires a switch from the traditional viewpoint. Instead of focusing on individual journeys from origins to destinations, it focuses on the analysis of what happens at the final destinations.

A similar approach can be used in p -hub center, hub covering, and p -hub median problems. Kara and Tansel (1999, 2000) have shown that a dramatic reduction in computation times has been achieved in the p -hub center and the hub covering problems by means of a similar change of variables. A similar change of variables also leads to a substantial reduction in CPU times in the p -hub median problem. Note, however, that the p -hub median problem has a multicommodity flow structure, which has been used in a clever way by Ernst and Krishnamoorthy (1996) for better solution times.

4. Time Zones

It is possible to have different time zones within the service area of a cargo delivery firm. Hall (1989) gives an extensive discussion of how different time zones affect flight arrival and departure times in air travel (see also Grove and O'Kelly 1986 and O'Kelly and Lao 1991). The incorporation of time zones is particularly important for cargo delivery firms that operate in a wide geographical area, e.g., international firms or national firms in North America. Most such firms promise to deliver cargo by a certain deadline expressed in the local time of the destination. In a large hub network, planes flying east will lose time and planes flying west will gain time from crossing time zones. In this section, we focus on this issue and present the appropriate modification to our model to correctly handle the effects of time zones. It will be evident from the discussion that this modification does not change the structure of the model and hence the modified model is as efficiently solvable as the Model L2 developed for a single time zone.

To handle the time zones, we follow the standard time zone convention of the U.S. Naval Observatory, Astronomical Applications (<http://aa.usno.navy.mil/AA/faq/docs/world-tzones.html>). In this convention, the world is divided into 37 time zones

numbered from -12 to $+14$ with time zone 0 referring to the Greenwich standard time and negative and positive numbers referring, respectively, to time zones west and east of Greenwich. Let $TZ_i \in \{-12, \dots, 0, \dots, +14\}$ denote the time zone of node i . Define Δ_{ij} to be the time gained or lost during the journey from node i to node j because of a change of time zone. That is, $\Delta_{ij} = TZ_j - TZ_i$ with $\Delta_{ij} > 0$ (< 0) if j is east (west) of i . Recall from (14) and (15) that \widehat{DT}_h and DT_h are the departure times from hub h toward other hubs and toward nonhub destinations, respectively. To incorporate the effects of time zones into the model, we redefine \widehat{DT}_h and DT_h as follows:

$$\widehat{DT}_h = \max_{k:a(k)=h} (r_k + t_{kh} + \Delta_{kh}), \quad (14')$$

$$DT_h = \max_{k \in H} (\widehat{DT}_k + \alpha t_{kh} + \Delta_{kh}). \quad (15')$$

With the presence of the addend Δ_{kh} in both definitions, the departure times are now adjusted to the local time at hub h . For example, consider a flight from New York to Istanbul. $TZ_{NewYork} = -5$ and $TZ_{Istanbul} = +2$, hence, $\Delta_{NewYork,Istanbul} = +7$. Consequently, 7 hours must be added to the trip time to compute the local arrival time at Istanbul.

An integer programming formulation for the problem with different time zones is as follows:

$$(L2') \min Z,$$

$$\text{s.t. } Z \geq DT_h + \rho_h \quad \forall h, \quad (26)$$

$$\rho_h \geq (t_{kh} + \Delta_{hk})X_{kh} \quad \forall k, h, \quad (27')$$

$$\widehat{DT}_k \geq (r_j + t_{jk} + \Delta_{jk})X_{jk} \quad \forall j, k, \quad (17')$$

$$DT_k \geq \widehat{DT}_r + (\alpha t_{rk} + \Delta_{rk})X_{rk} \quad \forall r, k, \quad (18')$$

$$(19) - (22).$$

(17') and (18') ensure that \widehat{DT}_k and DT_k take on the intended values as defined in (14') and (15'). The addend Δ_{hk} in the right-hand side of (27') adjusts the flight time from hub h to node k according to the local time at node k . We refer to the above model as $L2'$. Observe that $L2$ and $L2'$ are essentially the same linear programs. Only the coefficients of the variables are slightly different due to the presence of the constants Δ_{ij} .

5. Computational Results

When we test the computational performance of the two linear models L1 and L2, we observe that the solution times for L2 are generally two to three times faster than those of L1. For this reason, we report our computational results for only L2. We use the standard CAB data set (O'Kelly 1987) for computational tests and use CPLEX 5.0 on a 8 CPU, 50 Mhz super Spar station with 384GB memory to solve 60 instances of L2. The CAB data set is generated from the Civil Aeronautics Board Survey of 1970 air passenger travel data in the United States. It provides the passenger flows and distances between 25 cities. We generate a total of $4 \times 3 \times 5 = 60$ instances corresponding to all combinations of (n, p, α) , where $n \in \{10, 15, 20, 25\}$, $p \in \{2, 3, 4\}$, and $\alpha \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$. The four problem sizes corresponding to different n utilize the distance data for the first n cities in the CAB data set as the $T = [t_{ij}]$ matrix. We take $r_i = 0 \forall i$ for every instance. In our computational study, we assume that there are no differences in time zones, i.e., all $\Delta_{ij} = 0$. However, an example that takes the time zones into account is discussed at the end of this section to highlight some effects of time zones on the solutions.

In Table 1, we provide the CPU seconds reported by CPLEX together with the optimal hub locations and objective function values for each (n, p, α) combination. As expected, the solution times increase with increased n and increased p . For example, in going from $n = 10$ to $n = 25$, the average solution time increases by 112 times for $p = 2$, while the increase for $p = 3$ and 4 are, respectively, 380 times and 1,263 times. As can be seen from Table 1, the discrepancy between average and maximum times reported for the same combinations of (n, p) are not too great. The maximum time never seems to exceed twice the average time. In 10 minutes, 45 out of 60 instances are solved to optimality. All instances, except those corresponding to $(n, p) = (25, 4)$, are solved within one hour, while the most difficult instances corresponding to $(n, p) = (25, 4)$ are solved in four and a half hours.

The reported times in Table 1 show that L2 is a successful linear integer formulation for solving all sizes of the standard test problems.

In general, the optimal objective value, Z_p^* is a non-increasing function of p (since the availability of more

hubs cannot increase the latest arrival time). This observation is confirmed from the Z_p^* values in Table 1 for each fixed (n, α) . An additional noteworthy observation, based on the data in Table 1, is that the ratio $(Z_p^* - Z_{p+1}^*)/Z_p^*$, which measures the relative decrease in the objective function value as p is increased by one unit, declines (for fixed p) as α is increased. For example, for $n = 10$, $(Z_2^* - Z_3^*)/Z_2^* = 0.27$ for $\alpha = 0.4$, while that ratio goes down to 0.21 when $\alpha = 0.6$. Similarly, again for $n = 10$, $(Z_3^* - Z_4^*)/Z_3^* = 0.18$ for $\alpha = 0.4$, and 0.11 for $\alpha = 0.6$. This ratio tends to 0 when α approaches 1. If these ratios are computed for the remaining values of n , the $(Z_2^* - Z_3^*)/Z_2^*$ is in the range between $[0.004, 0.27]$, while $(Z_3^* - Z_4^*)/Z_3^*$ is in the range $[0, 0.26]$. Hence, the largest reduction that can be expected from a unit increase in the number of hubs is about 27%. What this implies is that, for a cargo delivery firm that imposes a certain deadline on delivery time (e.g., 8:30 a.m. the next morning), if the optimal latest arrival time for a given value of p exceeds this deadline by more than 27%, then the firm should think about increasing the number of hubs by at least two to come closer to meeting the deadline.

We now focus on the effects of the parameter α on the structure of the locations of hub nodes and the allocations. The general conclusion is that, when α is changed in the range between 0.4 to 1, the location and allocation decisions are unaffected for most values of n and p , while these decisions are more sensitive to changes in α for values of α smaller than 0.4. For example, in 7 out of the 12 possible combinations of (n, p) , the locations of hub nodes and the allocations of nonhub nodes to the hubs remain unchanged when α is increased from 0.6 to 0.8. In the remaining 5 combinations, the hub sets for $\alpha = 0.6$ and 0.8 differ by only one node. The insensitivity of the solution to α is even more evident for a larger range of α for the (n, p) combinations (10, 3) and (15, 3). For these combinations, the locations and allocations remain unchanged when α is in the range from 0.4 to 1. The sensitivity of the solution to changes in α when $\alpha \leq 0.4$ is striking for the case $(n, p) = (25, 3)$. For this case, the location and allocation decisions for $\alpha = 0.2$ are completely different from those for $\alpha = 0.4$.

It is interesting also to investigate possible effects of α on the interhub distance. We illustrate these effects

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Table 1 CAB Data Results

<i>n</i>	α	<i>p</i>								
		2			3			4		
		CPU	Z_p^*	Hubs	CPU	Z_p^*	Hubs	CPU	Z_p^*	Hubs
10	0.2	3.7	1425.0	6, 7	5.5	1118.0	6, 8, 10	6.9	831.2	2, 5, 7, 8
	0.4	3.7	1627.0	6, 7	5.9	1185.0	6, 7, 8	7.5	969.2	3, 5, 7, 8
	0.6	2.9	1758.0	5, 8	5.0	1387.0	6, 7, 8	9.9	1147.8	3, 5, 7, 8
	0.8	3.6	1758.0	5, 8	4.7	1589.0	6, 7, 8	12.5	1456.4	3, 5, 7, 8
	1.0	2.3	1839.0	4, 10	7.5	1791.0	6, 7, 8	6.3	1766.0	3, 4, 7, 8
	Avg.	3.2			5.7			8.6		
	Max.	3.7			7.5			12.5		
15	0.2	19.0	2003.2	5, 8	33.3	1750.0	1, 6, 8	100.8	1341.2	6, 11, 12, 14
	0.4	21.0	2162.0	5, 12	63.4	1760.0	3, 12, 13	71.6	1435.4	2, 11, 12, 14
	0.6	11.0	2213.8	5, 12	25.7	1843.6	3, 12, 13	199.7	1755.4	1, 3, 11, 12
	0.8	23.0	2423.8	12, 13	30.4	2164.8	3, 12, 13	145.8	2080.0	3, 11, 12, 14
	1.0	9.8	2611.0	11, 12	70.1	2600.0	3, 12, 13	200.6	2600.0	4, 10, 11, 12
	Avg.	17.0			44.6			143.7		
	Max.	23.0			70.1			200.6		
20	0.2	109.0	1892.0	1, 19	347.0	1548.2	9, 16, 19	1302.0	1355.8	11, 14, 19, 20
	0.4	102.0	2162.0	5, 19	524.0	1760.0	3, 18, 19	2653.0	1472.4	11, 14, 18, 19
	0.6	108.0	2278.0	13, 19	465.0	1995.8	9, 13, 19	691.3	1834.8	11, 14, 17, 19
	0.8	59.0	2507.4	13, 19	527.0	2263.4	3, 17, 19	2268.0	2153.4	11, 12, 14, 17
	1.0	79.0	2611.0	11, 12	495.0	2600.0	9, 11, 12	2007.0	2600.0	4, 12, 13, 19
	Avg.	91.0			472.0			1784.0		
	Max.	109.0			527.0			2653.0		
25	0.2	434.0	2136.0	21, 22	1642.0	1912.8	2, 13, 22	7252.0	1616.2	9, 16, 19, 23
	0.4	417.0	2400.4	5, 8	2653.0	2098.2	1, 6, 8	10869.0	1880.4	3, 12, 13, 23
	0.6	432.0	2556.6	8, 21	1882.0	2335.2	8, 9, 16	6014.0	2183.2	19, 21, 22, 23
	0.8	243.0	2712.8	8, 21	2206.0	2551.6	6, 8, 16	15906.0	2456.8	19, 21, 22, 23
	1.0	362.0	2826.0	8, 11	2398.0	2762.0	8, 11, 23	13224.0	2726.0	4, 8, 23, 24
	Avg.	378.0			2156.0			10653.0		
	Max.	434.0			2398.0			15906.0		

using a specific example with $n = 20$, $p = 2$. For this combination of n and p , the optimal hub locations for $\alpha = 0.2$ are Phoenix and Atlanta with Phoenix serving Phoenix, Los Angeles, and Denver, and Atlanta serving the remaining cities. As α changes from 0.2 to 0.8, Phoenix stays as a hub, while the other hub moves from Atlanta to Cincinnati at $\alpha = 0.4$ and to Memphis at $\alpha = 0.6$ and 0.8. At $\alpha = 1.0$, the hub at Phoenix moves to a new hub at Los Angeles (farther west) and the one at Memphis moves to Kansas City. Meanwhile, the allocation set of the first hub, the one that is initially at Phoenix and later at Los Angeles, has

shrunk from the 3 cities Phoenix, Los Angeles, and Denver to a single city, Los Angeles, as α is increased from 0.2 to 1. The interhub distance has decreased as long as the first hub has stayed at Phoenix (i.e., as α is increased from 0.2 to 0.8), while it increased when α is changed from 0.8 to 1. Even though this pattern (hubs getting closer first and farther apart later) seems somewhat unusual, it is actually understandable when the structure of the problem is considered. There are two main factors affecting the latest arrival time: (i) the interhub distances and (ii) the maximum city-to-hub travel time among the cities

each hub serves. For different parameter settings of the problem, either (i) or (ii) becomes dominant in defining the latest arrival time. In the CAB data set, we observe that, for $n = 20$ with $p = 2$, up to a certain value of α , the model reacts to the increases in α by getting the hubs closer. However, after that α level, to decrease the maximum city-to-hub travel time for each hub, increasing the interhub distance becomes more favorable.

Let us now focus on the question of achieving a fixed time commitment. This could be accomplished by either using faster aircraft (i.e., smaller α) or employing more hubs with the same aircraft. To investigate this issue, we take the case of $n = 10$ and order the objective function values corresponding to 15 combinations of α and p . This ordering gives us Z values of 831, 969, 1,118, 1,148, 1,185, 1,387, 1,425, 1,456, 1,589, 1,627, 1,758, 1,758, 1,766, 1,791, and 1,839 corresponding, respectively, to (α, p) combinations of (0.2, 4), (0.4, 4), (0.2, 3), (0.6, 4), (0.4, 3), (0.6, 3), (0.2, 2), (0.8, 4), (0.8, 3), (0.4, 2), (0.6, 2), (0.8, 2), (1.0, 4), (1.0, 3), and (1.0, 2). The ordered Z values can be grouped into six clusters where each cluster contains Z values that are reasonably close together. Using parentheses for each cluster, the six clusters we identify contain the following Z values: (831), (969), (1,118, 1,148, 1,185), (1,387, 1,425, 1,456), (1,589, 1,627), and (1,758, 1,758, 1,766, 1,791, 1,839). Observe that the least expensive (α, p) combination in the last cluster is (1.0, 2), achieving a Z value of 1,839. Increasing p to 3 and 4 while keeping the same α reduces Z to 1,791 and 1,766, respectively, which is probably not well justified in terms of the additional costs for opening new hubs. On the other hand, keeping the number of hubs at $p = 2$ while decreasing α to 0.8 reduces Z to 1,758, which is probably a less costly alternative than opening new hubs. Observe, however, that further reduction of α to 0.6 does not improve the Z value. A jump from the last cluster to the next-to-last cluster requires either a substantial reduction in α while keeping the same p (e.g., from (1.0, 2) to (0.4, 2), corresponding to a reduction in Z value from 1,839 to 1,627), or an increase in p accompanied by a minor reduction in α (e.g., from (1.0, 2) to (0.8, 3), corresponding to a reduction from 1,839 to 1,589). Similar behavior seems to be dominant at clusters corresponding to smaller values of Z , but

with somewhat lesser reductions in α . Similar conclusions can also be made based on the data of Table 1 corresponding to $n = 25$.

Focusing now on the optimal hub locations and the critical paths encountered in each of the 60 instances of the CAB data set, we observe that for each value of n , there is a "special" node that is either included in the optimal hub locations or in the critical path determining the latest arrival time, regardless of the value of p and α . For example, for $n = 10$, Node 8 is in the hub set for every (n, α) combination when $p \geq 3$. For $p = 2$, Node 8 is in the hub set for $\alpha = 0.6$ and 0.8, and it is an origin or a destination of the critical path for other α values. Similarly, for $n = 15$, Node 12 is in the hub set for 58 of the 60 instances. In the remaining 2 instances corresponding to $\alpha = 0.2$ with $p = 2$ and 3, Node 12 is again an origin or a destination of the critical paths. Similarly, this special node is Node 19 for $n = 20$ and Node 23 for $n = 25$. It appears that the special node is a most isolated node in most cases (an exception occurs, for example, for $n = 20$ with Node 19).

We now switch attention to time zones and investigate their effects on the model solution. An illustrative example corresponding to $n = 25$, $p = 2$, and $\alpha = 0.8$ reveals quite interesting structural changes in the solutions with and without time zones. To solve this instance, we first transform the distances in the CAB data set to travel times. Of the 300 pairs corresponding to upper triangular part of the 25×25 travel time matrix, 75 are directly available from the Web page of Delta Airlines. The travel times for the remaining 225 pairs are estimated via a least squares regression model based on the data of the 75 pairs. Solving the selected instance without paying attention to time zones places the two hubs at Denver and Cincinnati, while the same instance with time zones retains one hub at Cincinnati and places the other one at San Francisco. The change of one of the hubs from Denver to San Francisco shows that the expected eastward shift of a hub in response to time zones, observed by Hall (1989), does not seem to be valid in case of multiple hubs. Even though the two solutions with and without time zones have a hub in common, a dramatic change occurs in the allocation sets. Without time zones, the hub at Denver serves the West

Coast cities Seattle, San Francisco, and Los Angeles in addition to Phoenix, Denver, and Houston, while Cincinnati serves all cities east of Denver (except Houston). When time zones are included, the hub at Denver not only moves to San Francisco but its allocation set shrinks to a single city, itself. All other cities are served by the hub at Cincinnati. This is somewhat unusual since Seattle, Los Angeles, and Phoenix are much closer to the hub at San Francisco than to the one at Cincinnati. The allocation of any of those three cities to San Francisco strictly increases the latest arrival time.

6. Analysis of Departure Times

Due to the problem definition, cargoes from certain origins and/or cargoes destined to certain destinations are actually waiting at the hubs from the time they arrive at the hub until the departure time of the plane that they are loaded on. These waiting times can be considered as "slack times," and one may want to question the utilization of these slack times in response to delays that can be encountered during the whole cargo delivery process. Specifically, one may ask how much delay can be tolerated at a given hub without increasing the latest arrival time.

Let (H, a) be a given solution and $f(H, a)$ be the latest arrival time induced by (H, a) . Let us now focus on the question of how much delay can be tolerated on the departure times of a specific hub, say hub q , without increasing $f(H, a)$. Recall that there are two different departure times at hub q , DT_q and \widehat{DT}_q . The delay on DT_q will only affect the arrival time at the destinations that receive service from hub q . On the other hand, the delay on \widehat{DT}_q will possibly affect the DT_k ($k \neq q$). Let δ_q^{\max} and $\hat{\delta}_q^{\max}$ denote the maximum tolerable delays on DT_q and \widehat{DT}_q , respectively, without increasing $f(H, a)$. Observe that

$$\delta_q^{\max} = f(H, a) - DT_q - \rho_q,$$

$$\text{where } \rho_q = \max_{j: a(j)=q} t_{jq},$$

$$\text{and } \hat{\delta}_q^{\max} = \min_{k \in H} (f(H, a) - \rho_k - \widehat{DT}_q - \alpha t_{qk}). \quad (28)$$

We can conclude now that as long as the departure time from hub q to other hubs is no later than

$\widehat{DT}_q + \hat{\delta}_q^{\max}$, and as long as the departure time from hub q to nonhub destinations is no later than $DT_q + \delta_q^{\max}$, the maximum arrival time resulting from (H, a) will not be any later than $f(H, a)$. Observe that δ_q^{\max} and $\hat{\delta}_q^{\max}$ can be effectively utilized in crisis management in response to unexpected events causing delays in departure times. For example, if one of the aircraft at hub q destined to go to a nonhub destination is grounded because of a mechanical problem, the person in charge may assign another available aircraft for that flight that can be ready in δ_q^{\max} time units. In this case, the latest arrival time to that destination will not exceed the intended optimum value.

Applying Equation (28) to all nodes selected as hubs will provide the maximum tolerable delays at each hub, assuming that there was no delay at the rest of the hubs. Observe here that there is at least one hub k for which $\hat{\delta}_k^{\max} = 0$ and at least one hub k' for which $\delta_{k'}^{\max} = 0$. Any delay at one of these hubs increases the latest arrival time by the amount of delay. Note also that there are an origin $s \in \arg \max_{i: a(i)=k} t_{ik}$ and a destination d such that $a(d) = k'$ with $t_{k'd} = \rho_{k'}$, so that (s, k, k', d) forms a critical path that determines the latest arrival time by the relation $f(H, a) = r_s + t_{sk} + \alpha t_{kk'} + t_{k'd}$. If there is more than one k for which $\hat{\delta}_k^{\max} = 0$ or more than one k' for which $\delta_{k'}^{\max} = 0$, then each such pair (k, k') identifies a critical path. Note here that alternative critical paths are never encountered in the CAB data set. If $f(H, a)$ needs to be reduced for some reason, one way of doing this is to find a solution (H', a') for which $f(H', a') < f(H, a)$. If $f(H, a)$ is already optimal, then this way of reducing $f(H, a)$ is not possible. A less costly alternative that does not require a change in the given solution (H, a) is to focus on the critical paths induced by (H, a) and reduce their total journey times. This can be done by either setting the appropriate r_i s to earlier times or by decreasing the flight times by assigning faster aircraft to critical path segments. Hence, the model on hand allows one to perform a trade-off analysis between the cost of reducing the critical journey times and the benefits that would be obtained from the reduction of the latest arrival time. Such analysis may prove to be quite useful when it is desirable to reduce the latest arrival time without changing the current hub locations and the current allocations of nodes to hubs.

At any hub q there will be many aircraft ready to depart at their departure times, \widehat{DT}_k and DT_q . While sequencing the aircraft for departure, it is important that the aircraft that fly in the segments of the critical path should depart first. The sequencing of aircraft at \widehat{DT}_q is more important than sequencing at DT_q since the aircraft flying at \widehat{DT}_q may affect DT_k of other hubs. A smart strategy may be to sequence the aircraft flying from q to k in increasing order of δ_k^{\max} s.

Suppose now we allow delays at many different hubs. In this case, the delay at any given hub affects the tolerable limits on the delays at other hubs, and so the analysis of simultaneous delays must take these interdependencies into account. Let $\delta_k, \hat{\delta}_k, k \in H$, be the delays associated with hub k . The delays will not affect the latest arrival time only if they satisfy

$$\hat{\delta}_j + \delta_k \leq b_{jk} \quad \forall j, k \in H, \quad (29)$$

where $b_{jk} \equiv f(H, a) - \widehat{DT}_j - \alpha t_{jk} - \rho_k$. Equation (29) is a system of p^2 linear inequalities in the $2p$ variables $\hat{\delta}_k, \delta_k, k \in H$. Any feasible solution to the nonnegativity constraints $\hat{\delta}_k, \delta_k \geq 0, k \in H$, and (29) constitutes a collection of delays on departure times that does not increase the latest arrival time beyond its current value. Using the Inequality System (29), we may answer what-if questions that address simultaneous delays at different hubs, such as: What if airport a has to be shut down for 2 hours, beginning at time t , for example because of stormy weather, and what if the sorting operations at hub b have to be delayed for 3 hours because of equipment malfunction beginning at time t' ?

We now present an example for the analysis presented in this section. We use AP data set (Ernst and Krishnamoorthy 1998) for this example since that data set is for a cargo delivery firm, Australian Post. We use the 20-node subset of the set, which is available in the OR Library of www.ms.ic.ac.uk. We use the distances as the T matrix. The AP data set contains different scaling factors for the traveling time from a nonhub city to a hub and from a hub to a nonhub, as well as the scaling factor α between two hubs. We take the two additional scaling factors as 1. We take $p = 3$ and $\alpha = 0.4$. The optimum objective value is 39.81 time units. In the optimum solution, Nodes 2, 8,

and 13 are selected as hubs. The solution is summarized in Table 2.

The critical path is (1, 2, 8, 20) in either way. $\hat{\delta}_2^{\max}, \hat{\delta}_8^{\max}, \delta_2^{\max}$, and δ_8^{\max} are zero since hubs at 2 and 8 are in the critical path. From (28),

$$\begin{aligned} \hat{\delta}_{13}^{\max} &= \min(39.81 - 17.81 - 11.73 - 9.48, \\ &\quad 39.81 - 11.57 - 11.73 - 12.04) \\ &= 0.79, \end{aligned}$$

and $\delta_{13}^{\max} = 39.81 - 27.29 - 11.73 = 0.79$. Thus, the maximum tolerable delay at Hub 13 for the vehicles departing to other hubs and to final destinations is 0.79 time units. If the delay at Hub 13 is longer than 0.79, then the optimum objective increases and the critical path changes. Suppose that there is a delay of 0.95 time units at Node 13 affecting \widehat{DT}_{13} . Then $\widehat{DT}_{13} = 11.73 + 0.95 = 12.68$. We can determine the new DT_2^{new} and DT_8^{new} as follows:

$$DT_2^{\text{new}} = 28.24 + \max(0, 12.68 + 12.04 - 28.24) = 28.24$$

$$DT_8^{\text{new}} = 22 + \max(0, 12.68 + 9.48 - 22) = 22.16$$

In this case, the critical path changes to (17, 13, 8, 20) and the new objective value is $39.81 + (0.95 - 0.79) = 39.97$, assuming that the hub locations and allocations do not change.

7. Conclusion

In this paper, we identified a new problem that we call the latest arrival hub location problem. Even though the new problem is closely related to the traditionally studied hub location problems, it differs from them in one major way: It seeks to minimize the latest arrival time at destinations, and hence takes into explicit account the transient times at hubs in addition to flight times. This is a more realistic model for cargo delivery systems than its closest relative, the p -hub center problem. The transient times at hubs are defined by the departure times

Table 2 Solution of Example From AP Set

Hub	Serves Nodes	\widehat{DT}_k	DT_k
2	1, 2, 3	11.57	28.24
8	4, 6, 7, 8, 11, 12, 16, 20	17.81	22.00
13	5, 9, 10, 13, 14, 15, 17, 18, 19	11.73	27.29

less the flight arrival times. There are two different departure times associated with a hub: one corresponding to trips to nonhub destinations, the other corresponding to trips to other hubs. Both of these departure times are determined by the latest of the arrivals at the hub. A hub-to-hub flight provides service for only incoming flights from nonhub origins, whereas a hub-to-nonhub flight provides service for incoming flights from nonhub origins as well as from other hubs. Consequently, there is a certain interaction between hubs as reflected in the computation of these departure times. Despite its apparent complications in the initially conceived model, certain simplifications are obtained to derive a leaner model that seems to focus on what really happens at the final destinations, rather than what happens during individual trips between origin-destination pairs. Nonlinear and linear IP formulations of the model are also given. Having different time zones in the service area is also discussed, and the model is adjusted to capture the effects of different time zones. Computational results based on standard test data indicate that medium-sized problems (e.g., $n = 25$) can effectively be solved using standard optimization tools, e.g., CPLEX. Additional results are supplied for the analysis of maximum tolerable delays at hubs without increasing the latest arrival time. Related what-if questions are also discussed.

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